Second-harmonic generation of Raman scattered light in a plasma channel

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The experimental results of Krushelnick *et al.* on second-harmonic generation of Raman scattered light at 45° to the backscatter direction may be understood as follows. The intense short pulse laser produces a plasma wave via Raman backscattering. In the region of the wake, the plasma shrinks in size, and forms a narrow plasma channel, of a few plasma periods width. The channel breaks into bubbles via oscillating two-stream instability on a picosecond time scale. The Raman backscattered light propagating through the channel of bubbles generates a second harmonic, observed at oblique angles. [S1063-651X(96)01309-8]

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I. INTRODUCTION

Krushelnick *et al.* [1] have recently reported experimental results on second-harmonic generation of Raman scattered light. An intense short pulse laser, 1.06 μm, 1 ps, 2×10^{18} W/cm^2 of spot size $\sim$10 μm shines on a jet of hydrogen or helium. A detector with an acceptance angle of 22° is mounted at 45° to the direction of the backscatter. The frequency spectrum of the received light contains a signal at $\omega_0-\omega_p$ and another one at $2(\omega_0-\omega_p)$ where ω_p is the estimated plasma frequency at the laser focus and ω_0 is the frequency of the laser. In the backward direction they observed orders of magnitude stronger signal at $\omega_0-\omega_p$ than at 45°, however, there was no component at $2(\omega_0-\omega_p)$.

The laser pulse seems to produce a fully ionized plasma of density $\sim10^{19}$ cm^{-3} and an electron temperature a few tens of eV on a femtosecond time scale via tunnel ionization [2]. The electrons experience a strong radial ponderomotive force and are pushed out of the paraxial region within a time of the order of plasma period, ω_p^{-1}, forming a nonuniform electron cavitation [3,4]. Raman backscattering [5,6] has a large growth rate, hence, strong signals are observed at $\omega_0-\omega_p$ in the backward direction. The side scattering suffers strong convection losses [7] and may not grow in a channel of radius $r_0\sim c/\omega_p$. Even if it grows its level would be weak.

We propose a probable scenario for the generation of $2(\omega_0-\omega_p)$ signals at 45°. The Langmuir wave generated in the Raman process has small group velocity. As it emerges out of the laser pulse it cannot sustain electron cavitation and the quasistatic electron density profile shrinks. A new equilibrium is found where ponderomotive force due to the plasma wave is balanced by the space charge force. The plasma wave is unstable to oscillating two stream instability [7,8] (OTSI) producing short wavelength zero frequency density perturbation. The Raman backscattered light propagating through an axially and radially nonuniform electron channel produces $\omega_0-\omega_p$ scattered radiation at oblique angles. The backscattered light also exerts $2(\omega_0-\omega_p)$ ponderomotive force on the electrons producing current density $\vec{J}_{2(\omega_0-\omega_p)}$. When $\vec{\nabla}\times\vec{J}_{2(\omega_0-\omega_p)}\neq0$, the current density produces $2(\omega_0-\omega_p)$ radiation at oblique angles.

In the underdense plasma, the wave vectors of Raman backscatter and second harmonic at 45° to the backscatter are $\vec{k}_1\simeq-\omega_0\hat{\vec{z}}/c$ and $\vec{k}_2\simeq\sqrt{2}(\omega_0/c)(\hat{\vec{x}}-\hat{\vec{z}})$, respectively. The electron density should have a Fourier component $\vec{k}_s=\vec{k}_2-2\vec{k}_1\simeq[\sqrt{2}\hat{\vec{x}}+(2-\sqrt{2})\hat{\vec{z}}]\omega_0/c$ to conserve momentum in the harmonic process. This implies that the radial and axial density scale lengths L_r,L_z should be of the order of c/ω_0. For $\omega_p/\omega_0=0.1$, $v_{\rm th}/c\sim10^{-2}$, where $v_{\rm th}$ is the electron thermal speed, these values of L_r,L_z are of the order of ten Debye lengths.

In Sec. II we estimate radial and axial extents of electron density channels in the wake region of the laser pulse. In Sec. III, the $2(\omega_0-\omega_p)$ signal at 45° to the backscatter is obtained. The results are discussed in Sec. IV.

II. QUASISTATIC DENSITY CHANNEL

Consider the propagation of an intense short pulse laser of pulse duration τ in an underdense plasma of electron density n_0^0 and electron temperature T_e,

$$\vec{E}_0=\hat{\vec{x}}E_0(r,t)e^{-i(\omega_0t-k_0z)},\tag{1}$$

where $k_0=(\omega_0^2-\omega_p^2)^{1/2}/c$, $\omega_p=(4\pi n_0^0e^2/m)^{1/2}$, and $-e$ and m are electronic charge and mass. The laser produces an oscillatory velocity of electrons $\vec{v}_0\simeq e\vec{E}_0/mi\omega_0$ and exerts a quasistatic ponderomotive force $\vec{F}_{\rm ps}=e\vec{\nabla}\phi_{\rm ps}$, where $\phi_{\rm ps}=m|v_0|^2/2e$ on them.

The ponderomotive force imparts quasistatic velocity to electrons and modifies their density to $n_0^0+n_s$, producing electrostatic field $\vec{E}_s=-\vec{\nabla}\phi_s$. Using the equations of motion and continuity for electrons, and employing Poisson's equation one obtains

$$\frac{\partial^2n_s}{\partial t^2}+\omega_p^2n_s=-\frac{n_0^0e}{m}\nabla^2\phi_{\rm ps}.\tag{2}$$

where $n_s\ll n_0^0$ has been assumed, electron thermal motions are neglected, and ion response has been ignored on a time

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scale much shorter than an ion plasma period, ω_{pi}^{-1}. On a time scale of a few electron plasma periods ($\omega_{pi}^{-1} \gg t > \omega_p^{-1}$) Eq. (2) gives

$$n_s \simeq -\frac{n_0^0 e}{m\omega_p^2}\nabla^2\phi_{\rm ps} \simeq -2n_0^0(1-r^2/r_0^2)|v_0|^2/r_0^2\omega_p^2, \quad (3)$$

where a Gaussian radial profile of the pump, $E_0^2 = E_0^2 \exp(-r^2/r_0^2)$, has been assumed. For $r_0 \sim c/\omega_p$ Eq. (3) gives $n_s/n_0^0 \sim -|v_0|^2/c^2$.

The laser pump undergoes stimulated Raman backscattering [5–7] (RBS), producing a plasma wave with potential $\phi = \phi(r)\exp[-i(\omega t - kz)]$ and an electromagnetic sideband wave, $\vec{E}_1 = \hat{x}E_1\exp[-i(\omega_1 t - k_1 z)]$ on a time scale [8] $\sim (c/|v_0|)(\omega_p\omega_0/2)^{-1/2}$, where $\omega_1 = \omega - \omega_0$, $k = k_0 + k_1 \sim 2\omega_0/c$. The group velocity of the plasma wave is low and the wave persists even after the laser pulse is gone. The oscillatory velocity of electrons due to the plasma wave v can take a maximum value of $\omega_p/k \approx (\omega_p/2\omega_0)c$ before wave breaking occurs.

The laser pulse is also susceptible to forward Raman scattering on a longer time scale of the order of $(c/|v_0|)\sqrt{2}/\omega_p$. The phase velocity of this wave is large $\sim c$ and it could grow to larger levels.

The large amplitude plasma wave is susceptible to oscillating two stream instability, producing a low frequency (or zero frequency) space charge perturbation $\phi' \simeq \phi' \exp[-i(\omega' t - k' z)]$ and sideband plasma waves $\phi_\pm \simeq \phi_\pm \exp[-i(\omega_\pm t - k_\pm z)]$ where $\omega_\pm = \omega' \pm \omega$ and $k_\pm = k' \pm k$. Following Liu and Tripathi [8], the ponderomotive force on electrons at (ω', k') due to ϕ and $\phi_\pm$ can be written as

$$\vec{F}_p' \equiv e\vec{\nabla}_p' \quad \text{where} \quad \phi_p' = (k_-\phi_- v/\omega_- + k_+\phi_+ v^*/\omega_+)/2.$$

The electron and ion density perturbations due to ϕ' and ϕ_p' are $n' = (k'^2/4\pi e)\chi_e'(\phi' + \phi_p')$,

$$n_i' = \frac{k'^2}{4\pi e}\chi_i'\phi',$$

where $\chi_e' \sim \omega_{pi}^2/k'^2 c_s^2$, $\chi_i' = -\omega_{pi}^2/\omega'^2$ are electron and ion subscriptibilities at ω', k', in the limit of $\omega' < k' v_{\rm th}$ and cold ions, where c_s is the sound velocity and ω_{pi} is the ion plasma frequency. Using n', n_i' in the Poisson's equation one obtains

$$\varepsilon'\phi' = -\chi_e'\phi_p', \quad (4)$$

where $\varepsilon' = 1 + \chi_e' + \chi_i'$.

The density perturbation n' couples with $\vec{v}$ to produce nonlinear density perturbations at $\omega_\pm$, $k_\pm$: $n_\pm^{\rm NL} = n' k_\pm p_\pm/2\omega_\pm$, where $p_+ = v$, $p_- = v^*$.

Using these in Poisson's equation we get

$$\varepsilon_\pm\phi_\pm = \frac{k^2}{d_\pm^2}(1+\chi_i')\frac{k_\pm p_\pm}{2\omega_\pm}\phi', \quad (5)$$

where $\varepsilon_\pm = 1 - (\omega_p^2 + k_\pm^2 v_{\rm th}^2)/\omega_\pm^2$. Equations (4) and (5) yield the nonlinear dispersion relation for OTSI:

$$\varepsilon' = -\frac{|v|^2}{4v_{\rm th}^2}\left(1 - \frac{\omega_{pi}^2}{\omega'^2}\right)\left(\frac{1}{\varepsilon_+} + \frac{1}{\varepsilon_-}\right). \quad (6)$$

For $k' > k$ and $\omega' > k' c_s$, ω_{pi} Eq. (6) simplifies to give the growth rate ($\gamma = -i\omega'$) of the maximally growing mode

$$\gamma = \omega_p |v|^2/8v_{\rm th}^2(1 + \omega_{pi}^2/k^2 c_s^2), \quad (7)$$

The wave number of the maximally growing mode is $k' \sim (\omega_p/v_{\rm th})[2\gamma/\omega_p]^{1/2}$.

For the parameters of the experiment k'^{-1} could be of the order of 1 μm. Thus the plasma wave in the region of the wake produces electron channels of radial width and axial periodicity of the order of ten Debye lengths.

III. SECOND-HARMONIC GENERATION

Consider the propagation of backscattered electromagnetic wave $\vec{E} = \hat{x}E_1\exp[-i(\omega_1 t - k_1 z)]$ through the region of the wake where quasistatic electron density can be taken as $n_0^0 + n'(r,z)$. The electromagnetic wave produces oscillatory velocity and oscillatory density, $\vec{v}_1 = e\vec{E}_1/mi\omega_1$, $n_1 = \vec{v}_1\cdot\vec{\nabla}n'/i\omega_1$. It also exerts a second-harmonic ponderomotive force $\vec{F}_{2p} = e\vec{\nabla}\phi_{2p}$ with $\phi_{2p} = -mv_1^2/2e$ on electrons, imparting on them an oscillatory velocity,

$$\vec{v}_2 = e(\vec{E}_2 - \vec{\nabla}\phi_{2p})/2im\omega_1, \quad (8)$$

where $\vec{E}_2$ is the self-consistent electric field of the second harmonic, $2(\omega_0 - \omega_p)$. The current density at $2(\omega_0 - \omega_p)$ is $\vec{J}_2 = \vec{J}_2^L + \vec{J}_2^{\rm NL}$, where $\vec{J}_2^L = -n_0^0 e\vec{E}_2/2im\omega_1$ and

$$\vec{J}_2^{\rm NL} = -(n_1\vec{v}_1 + n'\vec{v}_2)e/2 = -\frac{e\vec{v}_1\vec{v}_1}{2i\omega_1}\vec{\nabla}n' - \frac{n'e}{4i\omega_1}\vec{\nabla}v_1^2, \quad (9)$$

where a term containing $n_0^0\vec{\nabla}v_1^2$ has been dropped as it cannot produce radiation at oblique angles to the z axis. Using Eq. (9) in Maxwell's equations, one obtains the following equation for the magnetic field $\vec{B}_2$ of the second harmonic in an underdense plasma ($\omega_p^2 \ll \omega_0^2$):

$$\nabla^2\vec{B}_2 + \frac{4\omega_1^2}{c^2}\vec{B}_2 = -\frac{4\pi}{c}\vec{\nabla}\times\vec{J}_2^{\rm NL}. \quad (10)$$

We solve this equation in two dimensions, ignoring y dependence. For z dependence of the form $\exp(ik_{2z}z)$, Eq. (10) can be written as

$$-\frac{\partial^2}{\partial x^2}B_{2y} + k_{2x}^2 B_{2y} = \frac{2\pi e}{c\omega_1}(k_{2z} - k_1)v_1^2\frac{\partial n'}{\partial x}, \quad (11)$$

where $k_{2x} = (4\omega_1^2/c^2 - k_{2z}^2)^{1/2}$. For $n' = n_{s0}\exp(-x^2/x_0^2)$, Eq. (14) gives B_{2y}, outside the plasma,

$$B_{2y} \sim \pi^{3/2} \frac{e}{\omega_1 c} (k_{2z} - k_1) x_0 v_1^2 n_{s0} e^{-k_{2x}^2 x_0^2/4}. \qquad (12)$$

When $k_{2x} x_0 \sim 1$, the ratio of second-harmonic field at 45° to the field of the Raman backscatter at $(\omega_0 - \omega_p)$ is

$$\frac{B_{2y}}{E_1} = \frac{\omega_p^2}{\omega_0^2} \frac{n_{s0}}{n_0^0} \frac{v_1}{c},$$

which could be of the order of 10^{-4} for the parameters of the experiment, viz., $\omega_p/\omega_0 \sim 0.1$, $n_{s0}/n_0^0 \sim 0.1$, $v_1/c \sim 0.1$.

One may estimate the side scattered signal at $(\omega_0 - \omega_p)$ at 45° in a similar way. The nonlinear current responsible for this is $\vec{J}_1^{\mathrm{NL}} = -n' e v_{1x} \hat{x}$. Solving the wave equation one obtains

$$B_{1y} \simeq \frac{2\pi^{3/2} e}{c} n_{s0} \frac{k_{1z} v_1}{k_{1x}} x_0 e^{-k_x^2 x_0^2/4}.$$

$B_{1y}/E_1 \sim 10^{-3}$ for the parameters mentioned above.

In cylindrical geometry $\vec{B}_{2y}$ at the receiver will be down by a factor $\sim (x_0/d)^{1/2}$ where d is the distance of the receiver from the plasma. The intensity of the second harmonic at the receiver $P_2 = cB_{2y}^2/8\pi \sim 10^3$ W/cm^2 for $E_1/E_0 \sim 0.1$ and power density of the incident laser pulse $\sim 10^{18}$ W/cm^2.

IV. DISCUSSION

The second-harmonic Raman scatter at oblique angles appears to be produced in the wake region of the laser pulse. It is sensitive to radial density scale length x_0. For significant second harmonic $x_0 d$ should be around 10–15λ_D, i.e., 1–2 μm. Phase matching in the z direction requires electron density ripple with wavelength ~ 2 μm. It could be produced via OTSI of the plasma wave generated in Raman backscatter or Raman forward scatter on picosecond time scale. The ratio of electric fields of $(\omega_0 - \omega_p)$ and $2(\omega_0 - \omega_p)$ signals at 45° is estimated to be around 10 at 10^{18} W/cm^2.

The region of harmonic generation is a thin plasma cylinder of radium 1 μm, hence, the power radiated is low. This could be large if the main laser pulse breaks into small filaments via relativistic filamentation instability. However, the spot size of a filament is of the order of c/ω_p, which is roughly the radius of the laser beam in the experiment, hence, breakup of the laser into small filaments is unlikely to occur.

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[1] K. M. Krushelnick, A. Ting, A. Fisher, C. Manka, E. Esarey, and H. R. Burris, NRL Report No. NRL/MR/6795-95-7668, 1995 (unpublished).

[2] V. B. Gildenburg, A. V. Kim, V. A. Krupnov, V. E. Semenov, A. M. Sergeev, and N. A. Zharova, IEEE Trans. Plasma Sci. **21**, 34 (1993); L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1945 (1944) [Sov. Phys. JETP **20**, 1307 (1965)].

[3] G. Z. Sun, E. Ott, Y. C. Lee, and P. Guzdar, Phys. Fluids **30**, 526 (1987).

[4] P. Mora and T. M. Antonsen, Jr., Phys. Rev. E **53**, R2068 (1996).

[5] C. S. Liu, M. N. Rosenbluth, and R. B. White, Phys. Fluids **17**, 1211 (1974).

[6] Rousseaux, G. Malka, J. L. Miquel, F. Amiranoff, S. D. Baton, and Ph. Mounaix, Phys. Rev. Lett. **74**, 4655 (1995).

[7] C. S. Liu, Plasma Phys. **6**, 123 (1976).

[8] C. S. Liu and V. K. Tripathi, *Interaction of Electromagnetic Waves with Electron Beams and Plasmas* (World Scientific, Singapore, 1994).